Scenarios Generation, Regret Decisions and Linear Programming

Stefano Benati[1]

[1]Department of Sociology and Social Research
School of International Studies
University of Trento

April 19, 2018

Scenarios and Decision-Making

The typical outcome of a futuristic exercise is the selection of a scenario set $\Omega = \{\omega_1, \dots, \omega_n\}$ that the future may reserve to decision makers. But then:

Anticipation and Proactive attitude:

Question: What should decision makers do?

People, companies, institutions have alternatives can make profits of good and bad events: Is there a theory that can help to make a correct decision?

The History of Mathematical Decision Making in Operations Research distinguishes between decisions in conditions of:

- Full knowledge.
- Risky environments.
- Uncertainty.

Full knowledge decisions

Full knowledge means that all input problems are known with certainty:

- Profit and Cost parameters are known;
- No random variables are implied;
- There is no systemic reaction to the decisions;
- The decision maker has full control on the implementation;
- There are no scenarios.

Methodology:

Deterministic models of mathematical programming: $\max\{f(x) : x \in D\}$.

Application: Classroom scheduling in a University.

Pros and Cons: Problems with many variables can be solved: 160 courses times 20 classes times 30 times slots $=$ 96.000 variables can be managed by a personal PC.

Decisions under risk

The archetypical risky situation is choosing between lotteries. Here, probabilities (that are objective), are used to weight losses and gains.

Risky environments:

The outcome of a the decision depends on the "state of the world", that is, after the decision the world reveals its state showing off an occurrence $\omega_i \in \Omega$.

- Probabilities are assigned to occurrences;
- Occurrences are not caused by other decision makers;

Remark: Outcome $\leftarrow$ Decision $\times$ State of the World ($D \times \Omega$).

Von Neumann's Decision Maker

According to the axioms of Von Neumann, Morgenstern (1947) or Savage (1954):

Von Neumann's Decision Maker:

Decision makers are described **as if** they maximize their expected utility.

To have a normative tool, we should have:

- The structure of the Utility function: $u : D \times \Omega \to \mathbb{R}$;
- A probability measure P on Ω.

Pros and Cons: Optimal decisions can be calculated through mathematic programming (but the problem dimensionality can be an issue). But the utility function and the probabilities are hard to estimate.

Decisions under Uncertainty

Definition: Uncertainty is when we cannot really assign probabilities to events.

Example: Medium and long term evolution of social systems, climate change, technology (r)-evolution: they are bringing about opportunities, traps, on which we could:

- React.
- Anticipate.

Both activities require to make decision, both the opportunities/decisions space is larger in the latter case!

Assumption: Outcome $\leftarrow$ Decision $\times$ State of the World.

Open problem:

What is the decision rule that the decision maker should follow? Answer: Minimum regret.

Peak Flow Management

Let's make some example on how these decisions can be implemented.
This is a real example: Flood Management in Iowa City (Spence, Brown: Water Resources Research, 2016).

The model: Outcome $\leftarrow$ Decision $\times$ State of the World:

- Scenarios:
  - Increasing Peak Flows (R+)
  - Stationary Conditions (R=)
  - Decreasing Peak Flows (R-)
- Decisions:
  - Do nothing (N)
  - Reservoir re-operation (D)
  - Raise Embankment (E)

The Payoff Table

The following table is justified by the fact that every decision is optimal for some state of the world:

	R-	R=	R+
N	**1**	0	-1
D	0	**2**	1
E	-1	1	**4**

Von Neumann's decision maker's choice can be represented **as if** it:

- is characterized by one utility function, for example $u(x) = x$ (linear function representing risk-neutrality).
- assigns probabilities to states of the worlds, for example $Pr = [1/6, 1/3, 1/2]$.

Estimating Utilities

Expected Utility Calculation:

- u[raising embankments] =
  $= (1/6)\times(-1)+(1/3) \times 1 + (1/2) \times 4 = 13/6$;
- u[doing nothing] $= (1/6)\times 1+(1/3) \times 0 + (1/2) \times (-1) = -(1/3)$.

Therefore one should suggest: u[raising embankments] $>$ u[doing nothing].

It is difficult to extend this model to more complicated settings, that is, the ones in which:

- Multiple scenarios are involved.
- A continuous of decisions are available.

because $u(\cdot)$ and Pr are difficult (or impossible) to estimate.

Max-Min Optimal Decisions

Now, let's consider a popular decision method: **the Max-Min**.
The Max-Min works as follows:

- Calculate the worst-case scenario for every decision.
- Elect the decision with the best worst case.

	R-	R=	R+	worst-case
N	1	0	-1	**-1**
D	0	2	1	**0**
E	-1	1	4	**-1**

Therefore here the decision of D would be made, because:

$$m[D] = 0 = \max\{-1, 0, -1\}$$

Min Regret Decisions

Basic principle: When our decision turns out not to be optimal, we feel regret of having miss the best choice.
From the original table:

	R-	R=	R+
N	**1**	0	-1
D	0	**2**	1
E	-1	1	**4**

The regret table is:

	R-	R=	R+	regret
N	0	2	5	**5**
D	1	0	3	**3**
E	2	1	0	**2**

Then, we advocate the choice of E, because it minimize the maximum regret from making the wrong decisions.

Regret theory: who, when and why?

Main feature of Min-Regret decision:

- It tends to provide a solution that works pretty well in many scenarios;
- It is independent from probabilities and utilities.
- It is coherent with empiric results on real life decision makers (Kanheman, Tversky, 1979).

It originates in three independent paper published in 1982:

- Fishburn (Journal of Mathematical Psychology): mathematic and axiomatic elaboration.
- Bell (Operations Research): decision analytic consequences.
- Loomes, Sugden (The Economic Journal): interpretation and empirical works.

What is original: It is a theory that brings back sentiments and feelings into decisions.

Interpretation of regrets

Regret theory accommodates empirical choices, **Rejecting transitivity of choices**.

Consider the following example:

	R-	R=	R+
N	3	0	1
D	1	3	0
E	0	1	3

Making pair-wise comparisons we get:

- $D \succ N$ (regret when R= occurs)
- $E \succ D$ (regret when R+ occurs)
- $N \succ E$ (regret when R- occurs)

So that we have a cycle of pair-wise preferences, while the usual transitivity property would predict that, from conditions 1 and 2, $E \succ N$ (not the contrary).

Regret theory and Optimization

Optimal decisions depends on **scenarios and decisions**! There is a warning here! To use regret theory as an analytical method it is important:

- Make all scenarios explicit;
- Make all decisions explicit.

In Mathematic Programming Terminology: D: the set of the decisions, Ω: the set of the scenarios;

$$\text{disappointment} = d(x,\omega) = \left[\max_{y\in D} f(y,\omega)\right] - f(x,\omega)$$

$$\text{regret} = r(x) = \max_{\omega\in\Omega} d(x,\omega)$$

Optimal decision rule:

$$\min_{x\in D} r(x) = \min_{x\in D}\left[\max_{\omega\in\Omega}\left[\left[\max_{y\in D} f(y,\omega)\right] - f(x,\omega)\right]\right]$$

Conclusion

As a conclusion, we advocate the use of the min-regret decision rule, because:

- People are driven by sentiments and feelings, min-regret accounts for that.
- It is coherent with empiric decisions in real life settings.
- **but** the rejection of the transitivity axiom poses a challenge!